

Title: What's the Point? - A TI-92 Investigation

Brief Overview:

This learning unit investigates the properties of perpendicular bisectors of chords and their relationship to the center of the circle. Follow up and extension exercises relate this concept to that of inscribed polygons.

Links to Standards:

- **Mathematics as Problem Solving**
Students will demonstrate their ability to inscribe polygons in a circle through the discovery of the circumcenter.
- **Mathematics as Communication**
Students will formulate a mathematical definition and express generalizations discovered through their investigations. Students will share their conclusions orally and in writing.
- **Mathematics as Reasoning**
Students will make and test their conjectures through applications.
- **Mathematical Connections**
Students will transfer the construction of the circumcenter to the application of inscribing polygons.

Grade/Level:

Grades 9-12, Geometry

Prerequisite Knowledge:

Students should have the following skills:

- Working knowledge of the TI-92
- Knowledge of the relationship between a central angle and the measure of its intercepted arc
- Knowledge of basic terminology related to circles

Objectives:

Using the TI-92, students will:

- investigate the property of perpendicular bisectors of chords.
- determine the significance of the intersection point of the bisectors.
- use the definition of a circumcenter to inscribe quadrilaterals in a circle.
- investigate the properties of an inscribed quadrilateral.

Materials/Resources/Printed Materials:

- TI-92
- Handouts: What's the Point? - TI-92 Lab Procedures, Student Observation Sheet, Group Exploration Activity
- Arc Measurement macro (reference: TI-92 Exploration: 92 Geometric Explorations on the TI-92, Michael Keyton, page 37, Texas Instruments)

Development/Procedures:

- Students will construct an arc, form chords using three points on the arc, construct perpendicular bisectors, and determine the shape which is formed about the intersection point of the perpendicular bisectors by using reflection techniques.
- Using data collected, students will prove their conjecture in written form.
- Extending their discovery, students will apply their discovery through the inscribing of a quadrilateral in a circle.
- Teacher's Notes:
 - 1) Caution students to minimize size of the arc construction, so as to accomplish complete reflection of the arc on the calculator screen.
 - 2) Encourage students to follow direction sheet in conjunction with data sheet.
 - 3) Encourage students to drag measurement values to the right-hand side of the screen.
 - 4) Make sure to link students' calculators with an arc measurement macro which is referenced above (Initial objects are points A, B, C.)
 - 5) At teacher's discretion, calculator keystrokes for the Group Exploration Activity are optional.

Performance Assessment:

Students are to complete the Student Observation Handout, as well as the Group Exploration Activity and be prepared to share results with the class orally and/or in written form.

Extension/Follow Up:

Suggested extensions include the incorporation of compass and protractor constructions, as well as exploration into polygons with more than four sides. Additional reference to real-world applications: <http://www.mcs.drexel.edu/~crrres/Corinth/Stadium.html>.

Authors:

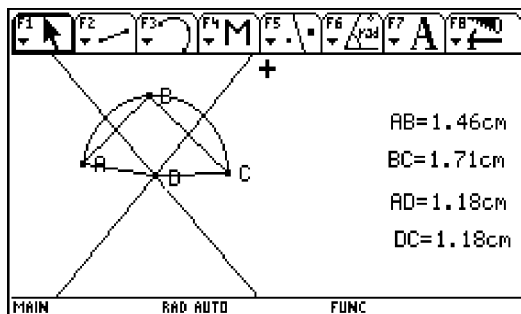
Lorraine C. Johnson
Annandale High School
Fairfax County, VA

Robert J. Kane
James Wood High School
Frederick County, VA

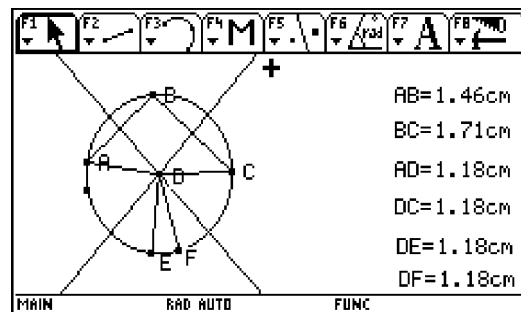
Catherine E. Meador
Granby High School
Norfolk City, VA

TEACHER'S NOTES

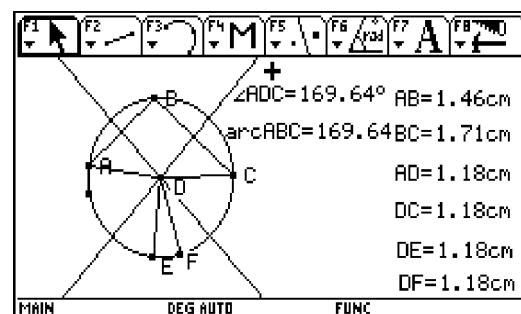
A collection of sample calculator screens are provided below to compare with student answers. Of course, measurements and sketches will vary depending on the students' constructions; however, the overall product should be comparable.



Sample screen of steps one through six.



Sample screen of steps one through nine.



Sample screen of steps one through eleven.

What's the Point?

TI-92 Lab Procedures

	Instructions	Keystrokes
1.	Construct arc ABC using the 3 point method.	[F3] , [2] , position pencil ... [ENTER] , A, move pencil, [ENTER] , B, move pencil, [ENTER] , C.
2.	Draw chords AB and BC.	[F2] , [5] , move pencil to A (THIS POINT), [ENTER] , move pencil to B (THIS POINT), [ENTER] , [ENTER] , move pencil to C (THIS POINT), [ENTER] .
3.	Construct perpendicular bisectors for each chord.	[F4] , [4] , position pencil on chord AB (PERPENDICULAR BISECTOR OF THIS SEGMENT), [ENTER] , repeat for chord BC.
4.	Plot a point at the bisectors point of intersection labeled D.	[F2] , [1] , position pencil (POINT AT THIS INTERSECTION), [ENTER] , D.
5.	Draw segments AD and CD.	Refer to step 2.
6.	Measure segments AB, BC, AD, CD. Move all measurements to the side as you find them. See the sample screen on the student observation sheet.	[F6] , [1] , position pencil at segment AB (LENGTH OF THIS SEGMENT), [ENTER] , AB=. Repeat for other segments.



Refer to Student Observation Sheet

	Instructions	Keystrokes
7.	Reflect arc ABC with respect to each of the perpendicular bisectors.	[F5], [4], move pencil to arc (REFLECT THIS ARC), [ENTER], move pencil to one of the bisectors (WITH RESPECT TO THIS LINE), [ENTER]. Continue in this manner until you have obtained a closed plane figure.
8.	Draw segments from point D to any two points on the new arcs.	[F2], [5], move pencil to D (THIS POINT), [ENTER], move pencil to point on arc (THIS POINT), [ENTER], [F7], [4], [ENTER], E, [ENTER]. Repeat to create segment DF.
9.	Measure segments DE and DF.	Refer to step 6.

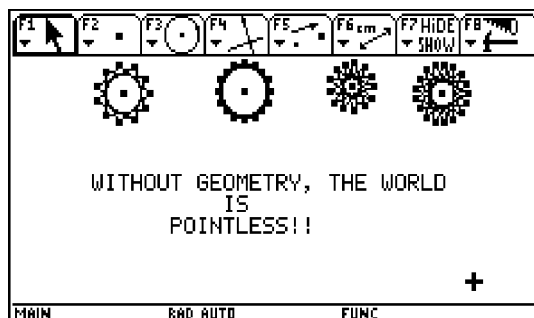


Refer to Student Observation Sheet

	Instructions	Keystrokes
10.	Measure $\angle ADC$	[F6], [3], move pencil to A (THIS POINT), [ENTER], then D (THIS POINT), [ENTER], then C (THIS POINT), [ENTER], $\angle ADC =$.
11.	Use arcmeas MACRO to measure arc ABC	[F8], [1] [F4], [6], [1], move pencil to A (THIS POINT), [ENTER], then B (THIS POINT), [ENTER], then C (THIS POINT), [ENTER]



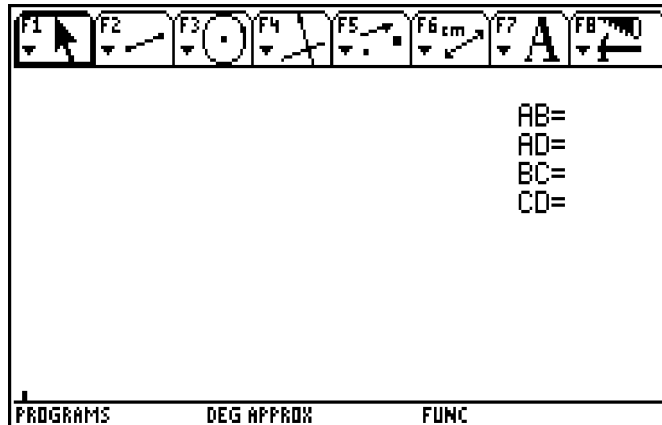
Refer to Student Observation Sheet.



STUDENT OBSERVATION SHEET

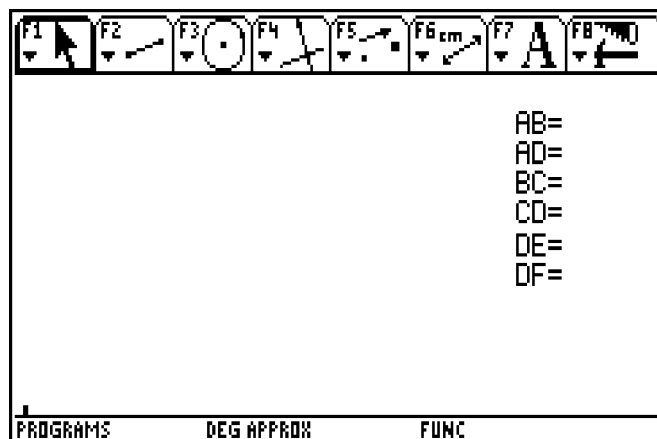
Complete this data sheet using the What's the Point? - TI-92 Lab Procedures handout. Be sure to answer each question completely using full sentences. When required, sketch your calculator screen in the space provided in detail.

1. At the completion of steps one through six, sketch your calculator screen below.



2. What do you notice with this construction? Be sure to include measurements of segments and make a prediction as to what you think is the significance of point D.

3. At the completion of steps seven through nine, sketch your new calculator screen below.

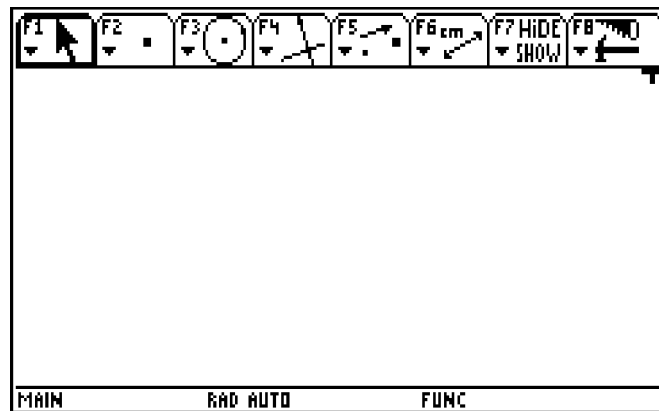


4. What does your picture tell you about point D? Be specific and defend your position.
5. Explain the significance of the results obtained in steps ten and eleven.
6. What is your final conclusion with respect to point D. Defend your position with specific information.

GROUP ACTIVITY EXPLORATION

In this activity you are to construct a non-regular quadrilateral and then construct a circle around it. Use your knowledge of perpendicular bisectors of chords to assist you in this activity.

1. Draw any four-sided non-regular polygon.
2. Construct the perpendicular bisectors of each side of the polygon.
3. What do you notice?
4. What should you do in order to be able to inscribe this polygon?
5. Try it. Sketch your calculator screen below.



6. Were you able to inscribe your polygon? Why or why not?
7. If so, what did you do? If not, what do you think you need to do?

8. If needed, try again until you have achieved success.
 - ☺ You may wish to check with your teacher before continuing.
 - ☺ Really!
9. Measure and record the four angles of your quadrilateral. What do you notice?
10. Do you think this relationship is true for all inscribed quadrilaterals? Defend your position.